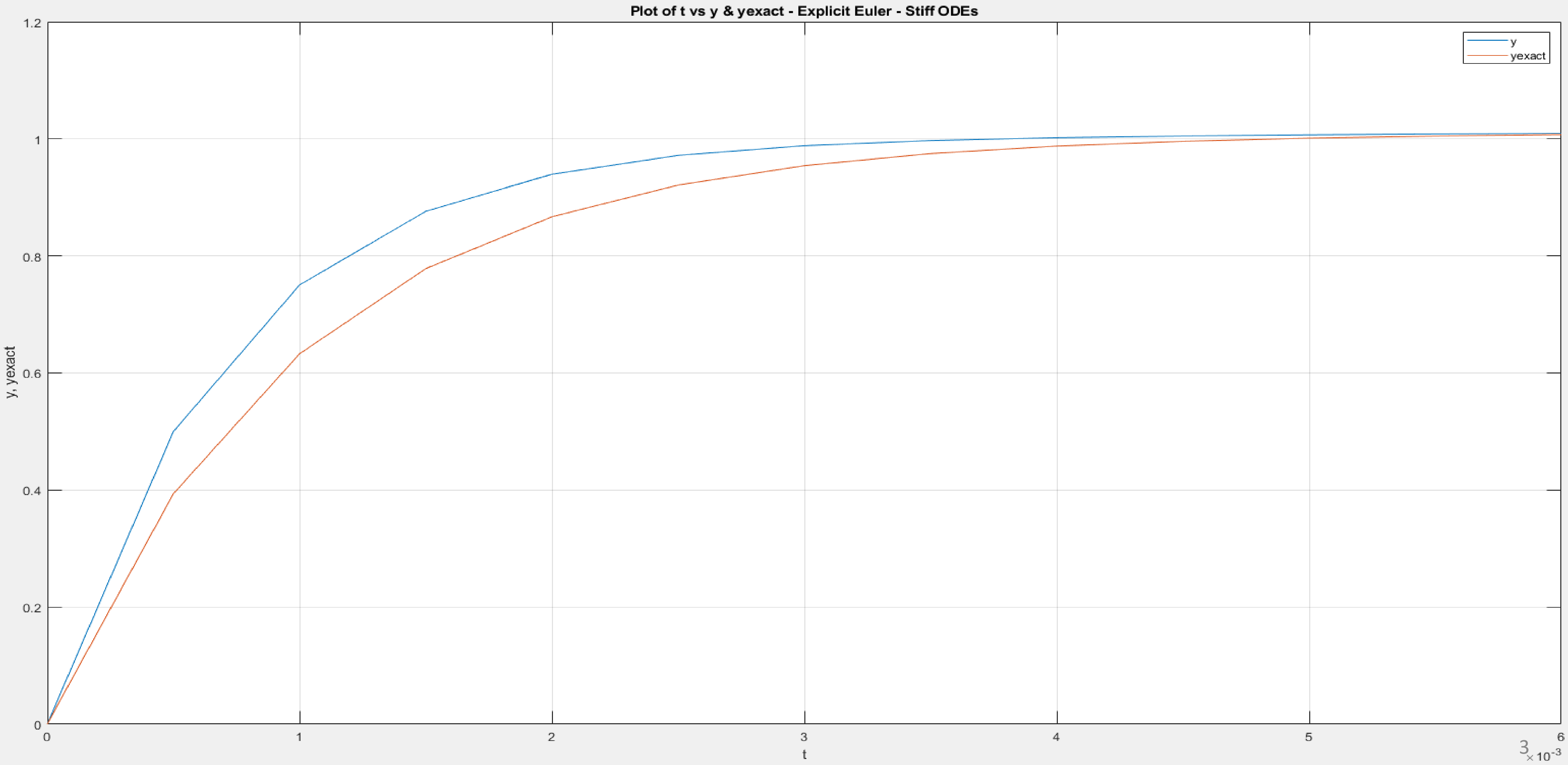
Solve Stiff ODEs Using Explicit, Implicit Euler Methods

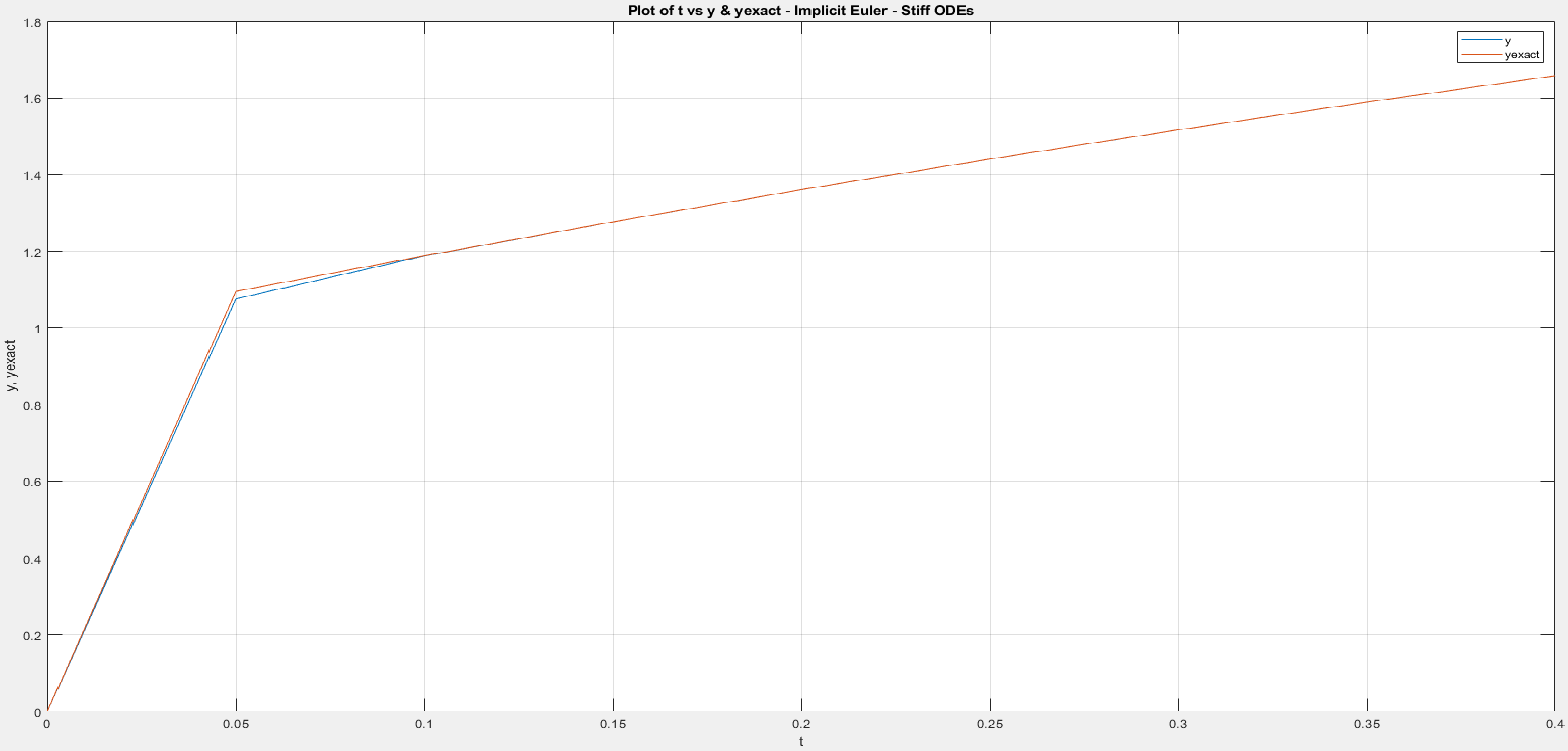
Introduction to Stiff ODEs

- Solve a stiff First order ODE (Initial Value Problem) Using Explicit and Implicit Euler Methods
- A stiff system is one involving rapidly changing components together with slowly changing ones
- An ODE is stiff if the step size required for stability is much smaller than the step size required for accuracy.
- An ODE is stiff if it contains some components of the solution that decay rapidly compared to other components of the solution.
- From a practical point of view, an ODE is stiff if the step size based on cost (i.e., computational time) is too large to obtain an accurate (i.e., stable) solution.

Solution for the Stiff ODE for small time

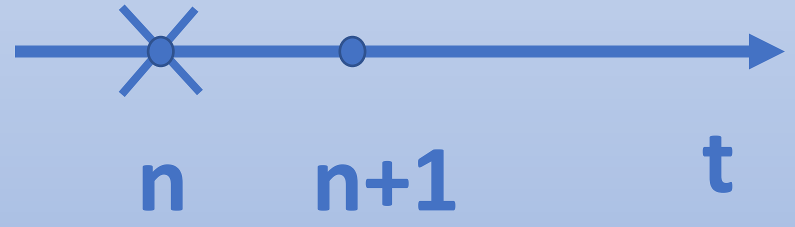


Solution for the Stiff ODE for Large times



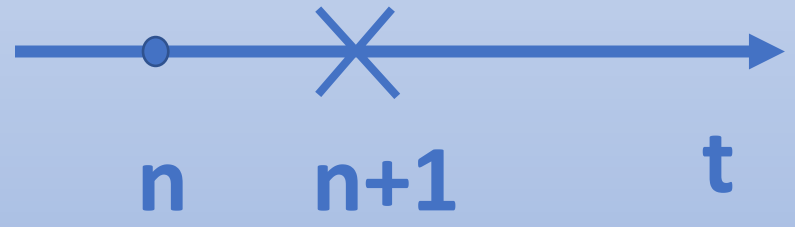
Explicit Euler Method

- Consider a general nonlinear first order ODE of the form
- $y' = f(t, y)$, $y(t_0) = y_0$ ....(1)
- $y_{n+1} = y_n + \Delta t * f_n$ $O(\Delta t)$ ....(2)
- Where $f_n = f(t_n, y_n)$
- Eq (2) is the FDE of the Explicit Euler's method
- Note n is the base point



Implicit Euler Method

- Consider a general nonlinear first order ODE of the form
- $y' = f(t, y)$, $y(t_0) = y_0$ ....(3)
- $y_{n+1} = y_n + \Delta t * f_{n+1}$ $O(\Delta t)$ ....(4)
- Let $f_{n+1} = f(t_{n+1}, y_{n+1})$
- Eq (4) is the FDE of the Implicit Euler's method
- Note n+1 is the base point



Solve Stiff ODEs Using Explicit Euler Method

- Example ODE Problem
- $\frac{dy}{dt}$ = - 1000 * y + 3000 – 2000 * e^{-t}
- From t = 0 to 0.006 with step sizes of 0.0005, 0.0015;
- The initial condition at *t* = 0 is *y* = 0.
- The exact solution is given as
- y = 3 – 0.998 * $e^{-1000 * t}$ - 2.002 * e^{-t}

Solve Stiff ODEs Using Explicit Euler Method

- Stability
- For an ODE of the form
- $\frac{dy}{dt}$ = - α * y
- Stability Criteria, Δt <= $\frac{2}{\alpha}$;
- For the equation, $\frac{dy}{dt}$ = - 1000 * y + 3000 – 2000 * e^{-t}
- Stability Criteria, Δt <= $\frac{2}{1000}$ <= 0.002

Solve Stiff ODEs Using Implicit Euler Method

- Example ODE Problem
- $\frac{dy}{dt}$ = - 1000 * y + 3000 – 2000 * e^{-t}
- From t = 0 to 0.4 with a step size of 0.05;
- The initial condition at *t* = 0 is *y* = 0.
- The exact solution is given as
- y = 3 – 0.998 * $e^{-1000 * t}$ - 2.002 * e^{-t}
- The solution is initially dominated by the fast exponential term $e^{-1000 * t}$. After a short period (*t* , 0.005), this transient dies out and the solution becomes dictated by the slow exponential e^{-t}

Summary

In this video,

- We presented Euler Explicit and Implicit Methods to solve a stiff Initial Value Problem ODE

- For a stiff problem, the step size required for stability is much smaller than the step size required for accuracy, if explicit methods are used.

- In the next video we can look at Finite Difference Method to solve Boundary Value Problem in ODE